Prof. Dr. M. Rapoport WS 2009/10

Program for the Seminar on Galois cohomology

worked out by G. Harder and M. Rapoport

1. Peter Scholze: Non-abelian Galois cohomology
Definitions and 'long' exact cohomology sequence ([11], I, §5.1–5.7). Forms ([11], III, §1). Use also the corresponding passages in [2] §§2, 3, and in [4], I, §7.

2. Raoul Blankertz: Cohomology of tori and class field theory
Explain Tate-Nakayama theory, cf. [4], III §2, cf. also [12], IX. Present the result also in terms of the dual torus, as in [9], §3. Perhaps mention the generalization to groups of multiplicative type, [11], II, §5.8.

3. Peter Scholze: p-adic fields
Prove in the case of classical groups Kneser's vanishing theorem for simply connected groups over a p-adic field and the surjectivity of the connecting homomorphism, cf. [4], IV., comp. also [14]. Deduce from this a classification theorem for reductive p-adic groups, cf. [9], §6.

4. Daniel Gerigk: Classification of reductive groups over $\mathbb{R}$
Explain [1], II, Satz 4, Thm 6, Satz 8, and III, Thm 8 and Thm 9, and give an instructive example from Kap. IV. Compare with the p-adic case.

5. Sean Wilson: Steinberg's Theorem
Explain the proof of Steinberg's theorem [13] which implies the vanishing of $H^1(k, G)$ for any connected linear algebraic group over a field k of cohomological dimension ≤ 1.

6. Timo Richarz: The Hasse principle, the proof for type A_n
Here the Hasse principle should be stated over a number field, as well as the surjectivity of the connecting homomorphism into $H^2(k, F)$, where F is the fundamental group of the adjoint group, cf. [4], V, §§1–3. Then the proof in the case of type A_n should be given, cf. [4], V, §5. See also [8]

7. Eugen Hellmann: The strong approximation theorem
The theorem should be explained and contrasted to the weak approximation theorem, comp. [5]. Then the proof should be explained in the case of the unit group of a simple algebra, cf. [6], §§3, 4. A proof in general that does not use the Hasse principle is in [10], §7.4.

8. Nicolas Vandenbergen: Landherr's theorem
This is the Hasse principle for type ${}^2A_n$. It should be proved, following [3], §2.

9. t.b.a.: The Hasse principle for some other types
Here one should explain how one can treat some other types, following [3], §3.

10. Paul Hamacher: Reformulation in terms of the L-group
Here one should give Kottwitz's formulation in [9], §4 (without using $Br_a(G)$).

It is not to be expected that each item above can be treated in one session of the seminar.

Literatur

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