Themenliste

Talk 01 – The Fourier Transform - Basics

Content: Sections 2.1 - 2.3

Abstract: We define the Fourier transform and discuss the basic properties. We will prove the Fourier inversion formula on the Schwartz class and introduce the notion of a Fourier multiplier.

Talk 02 – Tempered Distributions and Sobolev spaces

Content: Sections 2.4 - 2.7

Abstract: We extend the Fourier transform to the space of tempered distributions, for which we discuss the basic definitions and notion of convolution of tempered distributions. We will introduce the Sobolev and Bessel potential spaces as important function spaces.

Talk 03 – Symbol classes, basic properties and motivation of composition

Content: Sections 3.1 - 3.2

Abstract: We will introduce pseudodifferential operators and the space of symbols $S^m_{1,0}$. We will prove basic results and motivate the problem of composition of pseudodifferential operators.

Talk 04 – Oscillatory integrals and double symbols

Content: Sections 3.3 - 3.4

Abstract: On our road to the composition of pseudodifferential operators, we introduce oscillatory integrals and double symbols.

Talk 05 – Composition of pseudodifferential operators and applications

Content: Sections 3.5 - 3.6

Abstract: We finish the discussion of composition of pseudodifferential operators and discuss immediate consequences. We will discuss applications, in particular on elliptic pseudodifferential operators.

Talk 06 – Boundedness on $C_b^\infty(\mathbb{R}^n)$, uniqueness of the symbol and adjoints

Content: Sections 3.7 - 3.8

Abstract: We will show that pseudodifferential operators are bounded linear operators on $C_b^\infty(\mathbb{R}^n)$. Furthermore, we will show that the symbol of a pseudodifferential operator is uniquely determined on the Schwartz class and learn about the adjoint of a pseudodifferential operator.

Talk 07 – Boundedness on $L^2(\mathbb{R}^n)$

Content: Section 3.9

Abstract: We will show that pseudodifferential operators with symbols $p \in S^0_{1,0}$ can be extended to bounded operators on $L^2(\mathbb{R}^n)$.

Talk 08 – Calderon-Zygmund decomposition and Marcinkiewicz interpolation

Content: Theorem 4.8, Lemma 4.9, Corollary 4.11 and Theorem 4.12

Abstract: We introduce the Calderon-Zygmund decompostion of a function. Using this, we show that the Hardy-Littlewood maximal operator is bounded from $L^1(\mathbb{R}^n)$ to $L^1_{\text{weak}}(\mathbb{R}^n)$ and prove the Marcinkiewicz interpolation theorem.

Talk 09 – Singular integral operators

Content: Sections 4.1 - 4.2, Theorem 4.4 and Section 4.5

Abstract: We will introduce and motivate singular integral operators. With the aid of the Calderon-Zygmund decomposition, we show that under Hörmander condition, they are bounded operators from $L^1(\mathbb{R}^n)$ to $L^1_{\text{weak}}(\mathbb{R}^n)$ and discuss important examples.

Talk 10 – Mikhlin multiplier theorem

Content: Sections 4.6 and 5.3

Abstract: We discuss conditions under which Fourier multipliers induce bounded linear operators on $L^p(\mathbb{R}^n)$, both in the scalar as in the vector-valued case.

Talk 11 – Kernel representation of pseudodifferential operators

Content: Theorem 5.12 and Theorem 5.19

Abstract: We show that, for every pseudodifferential operator, there is a not necessarily translation invariant kernel and discuss properties of the kernel. As a consequence, we will see that pseudodifferential operators are bounded on $L^q(\mathbb{R}^n)$, $q \in (1, \infty)$.

Talk 12 – Besov spaces, Hölder spaces and Bessel potential spaces

Content: Sections 6.2 and 6.3

Abstract: We will introduce the Littlewood-Paley decomposition and its connection to Hölder spaces. We will learn about Besov spaces and discuss their connection to Bessel potential spaces.

Talk 13 – Applications of the Mikhlin multiplier theorem

Content: Section 7.1

Abstract: We will discuss applications of the Mikhlin multiplier theorem on the resolvent of the Laplace operator. We will study the spectrum of Fourier multipliers with homogeneous symbols, including the Laplace operator and, more generally, homogeneous constant coefficient differential operators.

Talk 14 – Applications of pseudodifferential operators

Content: Section 7.3

Abstract: We discuss regularity results for elliptic pseudodifferential operators and resolvents of parameter-elliptic differential operators.
