Minisymposium 3 - Stochastic Processes with Jumps: Theory and applications

On generalized coupled continuous time random walks

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The continuous time random walk (CTRW) model incorporates waiting times J_i between jumps Y_i of a particle. Classical assumptions are that $J_1, J_2, \ldots$ are iid belonging to some domain of attraction of a stable subordinator $D(t)$; $Y_1, Y_2, \ldots$ are iid belonging to the domain of attraction of an (operator) stable Lévy motion $A(t)$ and that (J_i) and (Y_i) are independent.
We present a two-fold generalization of this model by considering general triangular arrays $\Delta = \{(J_i^{(c)}, Y_i^{(c)}) : i \geq 1, c \geq 1\}$ of waiting times and jumps with iid rows and allowing arbitrary dependence between the waiting time $J_i^{(c)}$ before the jump $Y_i^{(c)}$. We assume that the row sums of Δ converge in distribution to some space-time Lévy process $\{(A(t), D(t))\}$. In this general setting the limiting distribution of the generalized CTRW modelled by Δ is of the form $M(t) = A(E(t))$ where $E(t)$ is the hitting time process of the subordinator $D(t)$, as in the classical case. However, since $A(t)$ and $D(t)$ are dependent, $A(t)$ and $E(t)$ are dependent. It turns out the the distribution of $M(t)$ can be represented in terms of $(A(t), D(t))$ even in this general coupled case. Moreover the Fourier-Laplace transform of the distribution of $M(t)$ is the solution to the so-called master equation in statistical physics. Finally the distribution of $M(t)$ is also the mild-solution of a coupled in space and time pseudo-differential equation generalizing fractional PDEs.